

Edith Cowan University  
**Research Online**

---

Theses : Honours

Theses

---

2000

## An analysis of option pricing under systematic consumption risk using GARCH

Alex Georgievski  
*Edith Cowan University*

Follow this and additional works at: [https://ro.ecu.edu.au/theses\\_hons](https://ro.ecu.edu.au/theses_hons)



Part of the [Econometrics Commons](#)

---

### Recommended Citation

Georgievski, A. (2000). *An analysis of option pricing under systematic consumption risk using GARCH*.  
[https://ro.ecu.edu.au/theses\\_hons/331](https://ro.ecu.edu.au/theses_hons/331)

This Thesis is posted at Research Online.  
[https://ro.ecu.edu.au/theses\\_hons/331](https://ro.ecu.edu.au/theses_hons/331)

# Edith Cowan University

## Copyright Warning

You may print or download ONE copy of this document for the purpose of your own research or study.

The University does not authorize you to copy, communicate or otherwise make available electronically to any other person any copyright material contained on this site.

You are reminded of the following:

- Copyright owners are entitled to take legal action against persons who infringe their copyright.
- A reproduction of material that is protected by copyright may be a copyright infringement. Where the reproduction of such material is done without attribution of authorship, with false attribution of authorship or the authorship is treated in a derogatory manner, this may be a breach of the author's moral rights contained in Part IX of the Copyright Act 1968 (Cth).
- Courts have the power to impose a wide range of civil and criminal sanctions for infringement of copyright, infringement of moral rights and other offences under the Copyright Act 1968 (Cth). Higher penalties may apply, and higher damages may be awarded, for offences and infringements involving the conversion of material into digital or electronic form.

# **An Analysis of Option Pricing Under Systematic Consumption Risk Using GARCH**

**Alex GEORGIEVSKI**

**November 2000**

---

**Submitted in partial fulfilment of the requirement for the degree of Bachelor of  
Business (Honours in Economics), Edith Cowan University.**

## USE OF THESIS

The Use of Thesis statement is not included in this version of the thesis.

### **Declaration:**

**This thesis contains no material which has been accepted for the award of any other degree or diploma in any tertiary institution and to the best of my knowledge and belief, the thesis contains no material previously published or written by another person, except when due reference is made in the text of the thesis.**

## **Abstract:**

We aim to test two things. Firstly, whether accounting for the persistence in volatility decreases the errors between the option prices implied from our models and the observed option prices and secondly, whether the pricing errors are reduced when you allow for the fact that consumption is correlated with returns on the underlying asset.

Three option pricing models are developed and tested. 1-The Black and Scholes option pricing model, 2-The GARCH (1,1) model under risk neutrality and 3- The GARCH (1,1) model under systematic consumption risk, using recent daily data on traded options on the FTSE 100 share price index.

Our findings suggest that when the persistence of the volatility of the underlying asset is accounted for, the pricing errors converge to the observed option prices ever so slightly, and only for certain options.

By allowing for systematic consumption risk, the implied option pricing model is more accurate than the other two models, but only for in-the-money call options. If the correlation between consumption and returns increases then this model will produce lower call option prices than the observed prices for in-the-money call options.

## **Acknowledgements**

I would like to thank the academic and administrative staff at the university for their support given to me without exception. Special thanks to Lurion DeMello for being available at all times with technical support. I neither knew nor expected that the journey this honours year would be one of self discovery, and I am glad it was not made alone. Lastly I would like to thank my supervisor Associate Professor Mansur Masih for inspiring the otherwise uninspired and for providing mentoring on a much grander landscape.

## **Contents**

1. Introduction	6
2. Conceptual Framework and Literature Review	10
3. Methodology	35
4. Data	51
5. Results	55
6. Summary and Conclusions	69
7. References	72



# I

## **Introduction.**

The persistent volatility of world stock markets since the 1997 Asian financial crisis is well documented. This paper looks at the implications of this persistent volatility for the pricing of traded options using recent daily data on the FTSE 100 stock index.

The options pricing literature is scattered but continually reverts back to the Black-Scholes (1973) model that assumes the underlying asset (the FTSE 100 in our case) has a constant variance. We put forward that this assumption does not match the evidence that the underlying variance changes through time.

The modelling of the variance is therefore crucial in the pricing of options. Evidence of time varying volatility can be seen in the graph of the FTSE 100 Log Returns shown in Figure One. Of note is the post 1997 period where prices and returns possess greater volatility. We use a GARCH process to model this persistent volatility as well as develop three conceptual models of option pricing.

Options are usually valued under risk neutral valuation, where investors' time preferences are constant. This assumption may not be valid so we

develop and test Rubenstein's (1976) model of option pricing thus allowing us to test the assumption of risk neutrality assumed under Black and Scholes. Our findings are that if consumption in the economy is correlated with the mean returns on stocks, then call options that are trading in the money will trade for less than the Black-Scholes and Risk Neutral implied models. The importance of modelling the variance and relaxing the assumption of risk neutrality is important as the Black-Scholes implied options prices are much higher for far-in-the-money options. This would imply that holders of the underlying asset (such as Fund Managers) receive a premium when they write options compared to the traded options on the market.

The paper is comprehensive in that it introduces option pricing in continuous and discrete time and makes use of Monte Carlo simulation techniques when no closed form solution to an option price is available, as is the case with stochastic volatility.

The three models developed are:

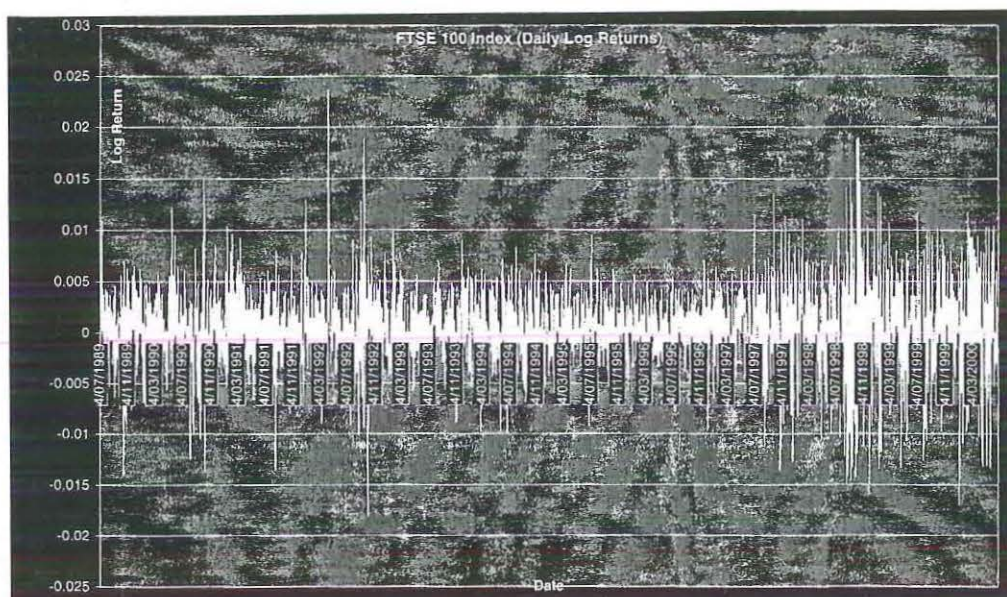
1-the Black-Scholes model,

2-the GARCH model under risk neutrality and

3-the GARCH model under systematic consumption risk.

When the steady state volatility increases then the prices implied by the three models should theoretically converge to be observed option prices. We test this convergence by observing the option prices at two different points in time where the steady state volatility is higher in the second period, post 1997. If the correlation between consumption and the mean returns on stocks increases, then the model which incorporates systematic consumption risk will underestimate call options which are in-the-money, when the steady state volatility increases. The other results simply confirm the regularities of option pricing models and basically cast further doubt upon the ability to find a model that successfully captures both stochastic volatility and

Section II develops the option pricing framework and presents a literature review. Section III outlines our methodology with an emphasis on Monte Carlo simulations. Section IV presents our data and the results are compiled in section V. A summary and conclusion complete our analysis in section VI.



**Figure 1: DAILY LOG RETURNS ON FTSE 100 INDEX. NOTE THE PERSISTENT VOLATILITY POST 1997.**

## **II**

### **Conceptual Framework and Literature Review.**

An option gives the holder the right but not the obligation to purchase (Call option) or sell (Put option) the asset underlying the option at a particular price (the exercise price) at some time in the future (expiry date). They are financial instruments, which are traded over the counter as well as on financial markets in all corners of the globe. We distinguish between a European option and an American option here, in that a European option is only exercisable on a certain date in the future, whilst the American option is exercisable upto a certain date in the future. We are only concerned with the pricing of plain vanilla European options in this paper, the pricing of American options as well as exotic type options including “barrier” and “knock-in knock-out” options is covered in numerous literature. (see “Black-Scholes and Beyond.” 1996). The use of options pricing in the valuation of different contingency claims and corporate liabilities is the focus of Smith (1976), where he uses options to value the debt and equity of a firm, to derive the Capital Asset Pricing Model (CAPM) and to value Coupon Bonds and Convertibles. Sarkar (1996) uses option pricing in the valuation of investment projects.

The Black-Scholes (1973) formula provides a method by which to price the simplest of contingent claims, that of European Puts and Calls. The method that Black-Scholes used to derive this formula was by the construction of a risk-less hedge by using a certain proportion of call options and the underlying asset. The argument was that because this hedge was “instantaneously” risk-less then the rate of return on this portfolio would be the risk-less rate if perfect substitutes yield the same rate of return. The call price is obtained from this equilibrium condition under the following assumptions:

- 1-No penalties for short sales
- 2-Zero Transaction costs and Taxes.
- 3-Continuously operating market.
- 4-Constant and known risk-less rate.
- 5-Underlying asset follows a continuous Ito process.
- 6-No dividends on stocks.
- 7-Exercise only on terminal date (ie European contract).

Merton (1973, 1976), Ingersoll (1976), and Cox and Ross (1976) discuss the implications of relaxing certain assumptions and derive hybrid models under relaxed assumptions. If we assume that these assumptions

do hold then the value of the hedge between a call option and the underlying asset will be given by:

$$V_H = Q_S S + Q_C C \quad (2.1)$$

$Q_S$  and  $Q_C$  refer to the amount of shares (the underlying) and amount of call options respectively, while  $S$  and  $C$  are the prices of the share and the call option respectively. Differentiating equation (2.1) gives us:

$$d V_H = Q_S d S + Q_C d C \quad (2.2)$$

The movement of the underlying asset, in this case the share price, will determine the change in the value of the hedge, assuming the quantities of both shares and call options do not change.

Assumption 5 states that the underlying asset (share price) follows a continuous Ito process. The call price can be seen as a function of the underlying asset price and time. Using Ito's Lemma, Black and Scholes derive the change in the call price  $dc$  as:

$$Dc = \partial c / \partial S dS + (\partial C / \partial t + 1/2 \partial^2 c / \partial S^2 \sigma^2 S^2) dt, \quad (2.3)$$

Notice that  $\sigma^2$  is the instantaneous variance of the underlying assets price and is constant through time. The only stochastic argument in equation (2.3) is  $\partial c / \partial S dS$ .

If the amount of shares and amount of calls were chosen so that  $Q_s / Q_c = -(\partial c / \partial S)$  then:

$$dV_H = Q_s Ds + Q_c [\partial c / \partial S dS + (\partial C / \partial t + 1/2 \partial^2 c / \partial S^2 \sigma^2 S^2) dt], \quad (2.4)$$

The first two terms in equation (2.4) equate to zero and are the only stochastic terms. This leaves:

$$dV_H = -(\partial C / \partial t + 1/2 \partial^2 c / \partial S^2 \sigma^2 S^2) dt, \quad (2.5)$$

As the hedge is risk-less and in equilibrium two perfect substitutes earn the same rate of return, the return to the hedge is equal to the risk-less rate:

$$dV_H / V_H = r dt. \quad (2.6)$$

Substituting equations (2.1) and (2.5) into equation (2.6) we have the change in the value of the call option through time:

$$\partial C / \partial t = rc - rs \partial c / \partial S - 1/2 \partial^2 c / \partial S^2 \sigma^2 S^2. \quad (2.7)$$



At the terminal date of a particular option, the call will be worth either the maximum between the stock price and the exercise price or zero. Smith (1976) defines this as the "boundary condition" for the solution of equation (2.7):

$$C = \text{MAX}[S - X, 0]. \quad (2.8)$$

This is a constrained maximisation problem in which we aim to maximise equation (2.7) subject to the constraint of equation (2.8).

$$C(S, X, \sigma^2, t, r) \quad (2.9)$$

The latent heat equation is sometimes used to draw the solution of the constrained maximisation problem but according to Smith (1977):

A more intuitive solution technique relies on the fact that, in describing the equilibrium return to the hedge, the sole assumption involving preferences of the economic agents in the market is that two assets which are perfect substitutes must earn the same rate of return: no assumptions involving risk preference have been employed. This suggests that if a solution to the problem can be found assuming a particular preference structure, then it must also be the solution to the differential equation for any other preference structure which permits a solution. Therefore, in solving the equation choose the preference structure which simplifies the mathematics. The simplest preference structure would be one in which all agents are risk neutral. In a risk neutral world the rate of return on all assets would be equal. Therefore, the current call price would be the expected terminal call price discounted to the present.

Risk neutral valuation of Black-Scholes is the second theme that we deal with in this paper. Just because risk neutral preferences simplify the mathematics in the Black-Scholes model does not imply that these preferences are the right ones in valuing contingent claims.

Black-Scholes make a further assumption that stock prices at a future date will have a log normal distribution:

$$C = e^{-rt} \int_X (S - X) L(S) dS, \quad (2.10)$$

In Equation (2.10) if  $L(S)$  is a log normal density function then the equation can be solved by assuming that with risk neutrality the average expected rate of growth is the risk-less rate  $r$ .

The solution to equation (2.10) and thus the solution to the European call option price is:

$$C = SN\{(\ln(S/X) + (r + \sigma^2/2)T)/\sigma\sqrt{T}\} e^{-rt} - XN\{(\ln(S/X) + (r + \sigma^2/2)T)/\sigma\sqrt{T}\} \quad (2.11)$$

$N\{\}$  is the cumulative standard normal distribution. Merton (1973) describes a solution to the European put pricing formula as follows:

$$P(S, T; X) = c(S, T; X) - S + XB(T) \quad (2.12)$$

Black and Scholes complete the solution as follows:

$$P = S N\left(\frac{-\ln(S/X) + (r + \sigma^2/2)T}{\sigma\sqrt{T}}\right) + X e^{-rt} N\left(\frac{\ln(S/X) + (r + \sigma^2/2)T}{\sigma\sqrt{T}}\right) \quad (2.13)$$

The only parameters in the equation are:

- 1-The price of the underlying.
- 2-The exercise price.
- 3-The time to maturity.
- 4-The variance of the underlying asset.
- 5-The risk-less rate.(the risk-free rate).

The Black-Scholes formula is a risk neutral valuation relationship (RNVRs). In this type of valuation relationship, the expected returns on all assets are assumed to be the same. Brennan (1979) describes RNVRs as depending only upon “potentially observable parameters”. These parameters are the ones just outlined:

An exact formula for an asset price, based on observable variables only, is a rare finding in a general equilibrium model.....

Merton (1973).

The Black-Scholes formula does not use investor preferences as a restriction, but assumes non-satiation. (ie more is always better). It does

assume continuous asset trading. The constrained maximisation solution described earlier to equations (2.7) and (2.8) will be preference free thus providing an RNVR. Cox and Ross (1976) define a resulting valuation relationship, as one where a constructed portfolio which consists of an option (contingent claim), and an underlying asset is in a proportion so that the instantaneous return on the portfolio is non-stochastic, a closed form solution.

A separate type of model can be described as a model of asset trading in discrete time intervals. It would not be possible to construct a portfolio with a return, which is non-stochastic. This type of model could describe the restrictions placed upon it by investor preferences. The Black-Scholes formula can also be derived in discrete time if on the aggregate level, all investors have utility functions which display constant proportional risk aversion, returns on the underlying follow a lognormal distribution, and the underlying asset is aggregate wealth. Rubenstein (1976) relaxes the final assumption so that the returns on the underlying and the returns on aggregate wealth (rate of growth of aggregate consumption), follow bivariate lognormal distributions. Discrete time models do not have the requirement that options and assets are to be continuously bought and sold. This allows the discrete model to be used more broadly in the valuation of many non-traded contingent claims, as described by Brennan

(1979), where he provides a single period model of market equilibrium which can be extended to a multi-period framework. There is no closed form solution in these models as there will be stochastic volatility.

There is numerous literature suggestive of stochastic volatility in the underlying asset of the option. Pagan (1996) provides a good summary of the econometrics of financial markets. Of particular importance to option pricing is his analysis of the clustering of volatility in high frequency financial series data. This clustering effect goes back to Hurst (1952), and the analogy to the clustering of the same sized diameters in the rings in trees. "Volatility in this respect will lead to further volatility". Capturing this persistence in the volatility of asset prices is therefore of paramount importance, as even under risk neutral valuation, the volatility of the asset underlying the option will affect the option price. As is evident in the Black-Scholes formula, the option price is a function of the variance of the underlying asset. If the underlying asset shows persistence in volatility, then the modelling of this persistence will be important in the pricing of the option at a particular point in time.

Engles (1982) ARCH, and Bollerslevs (1986) GARCH models are seen as the cornerstone in the modelling of the clustering of high frequency returns. These models are explained in section III.

Duan (1993) Presents the discrepancies between the Black-Scholes implied option prices and GARCH based prices and Engle and Mustafa (1992) assume that the GARCH (1,1) process is the risk-neutral distribution which describes the present value operator which is used to price options.

In our discrete time model (an arbitrage free economy) a risk-neutral probability distribution of payoffs from an option will exist. As Engle and Mustafa put it:

This distribution, otherwise known as the equivalent martingale measure, is derived by means of re-weighting the original probabilities of different states. The weight for each state is based on the intertemporal marginal rate of substitution (of the representative investor) between present consumption and consumption at that date in the future. In such an economy, the theoretical price a rational investor would pay for a European option on an asset with stochastic volatility is simply the expected value of the payoffs at the terminal date, discounted at the risk-free rate, where the expectation is taken using the risk-neutral probability function.

This is represented in the following equation:

$$c_t = e^{-n(T-t)} E_t[\max\{s_T - K, 0\}] \quad (2.14)$$

The expectation in equation (2.14) is taken under risk-neutrality. Under The constant variance of the underlying assumed in Black and Scholes, recursion's can be done on:

$$s_{t+1}/s_t = (1 + r_t) + \sigma_t \xi_{t+1} \quad (2.15)$$

Where:

$$\{\xi_t\} \sim i.i.d. N(0,1), \quad (2.16)$$

If a GARCH (1,1) stochastic process describes the expectation in equation (2.14) then we would have the following recursion instead:

$$s_{t+1}/s_t = (1 + r_t) + \sigma_{t+1} \xi_{t+1} \quad (2.17)$$

with the innovation in  $\xi_{t+1}$  being identically and independently normally distributed in accordance with equation (2.16) and the variance allowed to vary according to the GARCH (1,1) process:

$$\sigma_{t+1}^2 = \omega + \alpha \xi_t^2 \sigma_t^2 + \beta \sigma_t^2, \quad (2.18)$$

which sufficiently captures the persistence of the conditional volatility according to the parameters  $\omega$ ,  $\alpha$  and  $\beta$ . Note that Satchell and Timmermann's (1995) GARCH (1,1) process:

$$\sigma_{t+1}^2 = \omega + \alpha \xi_t^2 \sigma_t^2 + (\alpha + \beta) \sigma_t^2, \quad (2.19)$$

differs somewhat from Engle and Mustafa's GARCH (1,1) shown in equation (2.18). In equation (2.19) the persistence in volatility from one period to the next is represented by the summation of both coefficients whilst in equation (2.18) the persistence in volatility continues only

according to the  $\beta$  coefficient. All up, the difference is quite small in that the size of the  $\alpha$  coefficient. It should be noted though, on theoretical grounds that equation (2.19) counts two  $\alpha$  coefficients in the GARCH process .

Engle and Mustafa's (1993) implied risk-neutral measure of the S & P 500 stock index, they make reference to pricing errors:

If the recursion (equations (2.14) and equation (2.17) in our analysis) is correctly specified and  $\xi_t$  is perfectly unpredictable, the relevant information set will simply be past stock prices. In a more complex world, more sources of information could be useful for forecasting stock return volatilities. Let  $c_t^*$  be the observed market price of this option. Because the information set is potentially larger, the econometrician will see a pricing error,  $c_t^* - c_t = \eta_t$ , which could of course simply be a recording error in the timing or even the price of the option. In some cases, the current stock price should be a sufficient statistic for all the additional information which is useful in forecasting the future distribution of the stock price; however, here it is clear that it cannot be since any information about the timing of future volatility will affect the stock price and option price differently. Thus it is not surprising to find that the theoretical and observed options prices differ. See Day and Lewis (1992).

Our goal in this paper is finding which model possesses the lowest pricing errors in the determination of options prices by using the latest data on the FTSE 100. This data highlights the persistence of volatility in the underlying asset prices. Whether it is the Black-Scholes formula with constant variance, Engle and Mustafa's risk-neutralisation approach, or



the Satchell and Timmermann approach, which incorporates systematic consumption risk is what we aim to test.

To incorporate systematic consumption risk in the pricing of European options, we introduce Rubensteins (1976) one period market portfolio model.

$$\begin{aligned} \begin{matrix} Ln(X_t/X_{t-1}) \mid \\ Ln(C_t/C_{t-1}) \mid \Omega_{t-1} \end{matrix} &= \begin{pmatrix} \mu & \sigma_x^2, & \kappa \sigma_x \sigma_c \\ \mu & \kappa \sigma_x \sigma_c, & \sigma_c^2 \end{pmatrix} \end{aligned} \quad (2.20)$$

$X$  represents stock prices and  $C$  represents consumption. The conditional distribution is given on the right hand side of equation (2.20). A GARCH model with the relevant specification can be used to explain the changing variance represented by the subscript on the conditional variance of stock returns.

Satchell and Timmermanns (1995) representative agent framework describes  $N$  individuals in the market who possess “time additive extended utility functions with identical index parameter  $\beta$ ”:

$$U(C_0, C_1) = U(C_0) + U(C_1) = 1/1-\beta (C_0^{1-\beta}) + \rho/1-\beta (C_1^{1-\beta}) \quad (2.21)$$

The representative investor's rate of time preference is represented by  $\rho$ . A risk neutral investor will have a value of zero for  $\beta$  in the above equation and therefore this investors options will be valued under risk neutral valuation methods.

European call options can be priced by the following Euler' condition:

$$C(X_0, K) = \rho E\{Max[X_T - K, 0](C_1/C_0)^{-\beta}\} \quad (2.22)$$

This equation is comparable to equation (2.14) if we introduce the notion of a risk-free bond in the following:

$$e^{-\rho t} = E[\rho(C_1/C_0)^{-\beta}], \quad (2.23)$$

which is representative of the price of a unit of consumption as it is the first order condition. Rubenstein (1976) eliminates the parameters of the consumption data, as it is impossible to calculate daily consumption data. Satchell and Timmermann then derive this equation for the risk premium of a share in a model with stochastic volatility and systematic consumption risk which is not diversifiable:

$$(\mu - r_f - (\sigma_c \kappa^2 t)/2)t = -\ln(E[\exp(\sum_{j=1}^t (\sigma_j - \kappa \sigma_c) \epsilon_j)]) \quad (2.24)$$

If we were to set  $\kappa$  equal to zero and the time varying variance  $\sigma_j$  equal to a constant  $\sigma$ , then we would have the Black Scholes formula. If we were simply to set  $\kappa$  to zero and allow stochastic volatility then we would have Engle and Mustafa's recursive process shown in equation (2.14).

The risk premium of any index (including the FTSE 100 in our case) can be simulated by generating innovations in the  $\varepsilon_j$  term. The stochastic volatility process can be generated from the specified GARCH model. The value of  $\beta$  in the utility function given in equation (2.21) will define the risk premium of the index. ( $\sigma_c^2 = \beta^2 \sigma_\varepsilon^2$ ).

Satchell and Timmermann also derive the following formula for the price of a European call option:

$$O_0 = X_0 \exp(-r_f t - (\sigma_c^2 \kappa^2 t/2)) \times E(\exp(-\kappa \sigma_c \sum_{j=1}^t \varepsilon_j) \times \text{Max}(\exp(t\mu + \sum_{j=1}^t \sigma_j \varepsilon_j) - K/X_0, 0)) \quad (2.25)$$

Monte Carlo simulations are discussed in section III. Their use in this application is to come to a value of the preference parameter  $\kappa\sigma$  in equation (2.24). The value of  $\mu$  which is the return on the underlying asset, will feed back upon the option price via this value of  $\kappa\sigma$  according

to Satchell and Timmermanns contention.  $\mu$  is not visible under risk neutral valuation and therefore does not appear in the Black-Scholes option pricing formula.

Satchell and Timmermanns (1995) study shows that overall, the Black-Scholes formula is the least biased in regard to pricing errors. Incorporating consumption risk into the analysis actually produced the biggest bias of all, in a mean squared errors (MSE) sense. As they state:

Introduction of stochastic volatility based on the estimated values of a GARCH process from the underlying asset leads to a substantially higher bias in the option prices compared to the benchmark Black-Scholes model with constant volatility. It turns out that most of this difference is due to the use of a low value of the steady state volatility in the GARCH models ( $\sigma / 1 - \alpha - \beta$ ). Once the steady state variance of the models with stochastic volatility is close to the constant variance of the Black-Scholes model, the MSE of the three models converge.

This paper specifically tests two separate periods where the steady state volatility is higher in the second period (ie post Asian Financial Crisis) than the first period. If Satchell and Timmermanns reason for such a high bias is true then we should observe smaller biases between the observed option prices and the theoretical ones depicted by our three models, as their data is from the same source as this papers but of an earlier period (1989 – 1992).

Engle and Mustafa's (1992) exposition of option pricing under risk neutralization using GARCH uses "minimized sum of squares" to show that their "Implied ARCH" model produces smaller errors than the Black-Scholes formula. This is certainly not unconditional though, as in their case, the Black-Scholes formula is better with put options.

Close comparisons are hard to make between the results obtained here in this paper with Engle and Mustafa's as our distribution of consumption risk and returns is in log-normal form whilst that of Engle and Mustafa's is in normal level form. The purpose of Engle and Mustafa's survey of option priced over the 1987 stock market crash period, was to see if the options market correctly prices the volatility of the underlying asset (the S & P 500 in their case). Of note is their assertion that the options market did in fact anticipate the decrease in volatility after the 1987 crash according to the persistence of the GARCH parameters of the coefficients of the squared error terms and the conditional variance. Engle and Mustafa's model overpredicts call options and underpredicts put options. The time to maturity also affects the implied options in that pricing errors increase.

Amin and Ng (1993) derive a general option-pricing formula which is consistent with stochastic stock return variance, stochastic consumption

growth variance, stochastic interest rates, as well as a systematic component in the stock return variance. They also incorporate jump diffusion processes in the stock returns. Their consumption based equilibrium approach is also based upon Rubenstein (1976) and Brennan (1979) and is thus comparable to ours. The difference is that Amin and Ng's process is one of mean reversion rather than that of a typical GARCH process.

This mean reversion process can be thought of as the GARCH-M model.

The GARCH process itself has 3 assumptions placed upon it:

- 1- The conditional mean is time invariant to risk premium (constant risk premium).
- 2- The shocks to volatility in response to both good news and bad news are symmetric. In other words, it implies that the positive and negative shocks of equal size elicit an equal response from the market
- 3- The shocks to volatility are stationary, temporary and not permanent.

By relaxing the first assumption, we describe the GARCH(p,q)–IN-MEAN model:

$$y_t = \phi x_t + \gamma h_t + \varepsilon_t \quad (2.26)$$

$$h_t = \alpha_0 + \sum_{i=1}^p \alpha_i \varepsilon_{t-i}^2 + \sum_{i=1}^q \beta_i h_{t-i} \quad (2.27)$$

$$\varepsilon_{j,t} / \Omega_{t-1} \approx N(0, h_t), \quad (2.28)$$

$y_t$  is the excess return or risk premium,  $x_t$  is an exogenous, or predetermined, vector of variables,  $\varepsilon_t$  is a random error,  $h_t$  is the conditional variance of  $\varepsilon_t$ , and  $\Omega$  is the information set. This model allows for excess return  $y_t$  to be determined by the vector  $x_t$  own conditional variances. The conditional variance is linearly dependent on the past behaviour of the squared errors and a moving average of the past conditional variances. The use of squared error terms implies that if innovations have been large in absolute value, they are likely to be large also in the future. The basic difference to the GARCH model is that the coefficient  $\alpha_0$  is allowed to vary with time.

Relaxing the second assumption allows negative news to affect the return more so than positive news. This is evidence of the leverage effect in that the debt to equity of a firm will increase thus increasing the perceived

risk in holding the firms share. So the distribution of returns are seen to be positively skewed rather than normal .

Relaxation of the final assumption governing the GARCH (p,q) model will see that shocks to the system are permanent and do not die out. We talk of these later in this section with reference to Bollerslev (1996) and Baillie's (1998) models of permanent shocks to long memory volatility models.

The direction of the bias inherent in prices obtained from the Black Scholes model is different between stocks with a strong systematic variance component and those with a strong idiosyncratic variance component. Further, the effect of mean reversion in variance on option prices is dependent on whether the mean reversion is related to the interest rate or whether the mean reversion is for the idiosyncratic component that is not related to the interest rate.

Along with Satchell and Timmermann we may well agree that there is a mean reversion process apparent but we limit our scope to the GARCH process and assume that the restrictions outlined above do hold. The mean reversion is held constant by the choice of  $\mu$  and is adjusted to reflect the changing preferences of investors. Once again as reflected by  $\beta$



in their utility function. When the marginal utility of consumption is low, mean returns on shares will be high so there will be a positive correlation between consumption and the return on a risky asset.

Pricing errors have been discussed, in that all investors will not have the entire information available to them when making decisions apart from previous returns. There will therefore be an expected error between the prices we derive and the observed prices in the market. This problem is escalated in Satchell and Timmermann in that they also include in their study measurements of untraded options. To avoid any further bias in the pricing errors themselves, our data only includes traded options.

With regard to specific contracts, Satchell and Timmermann find that in the case of shorter maturity call options, the implied prices from the three option pricing models, all tended to be biased downwards compared to the quoted option prices. The Black-Scholes implied prices tended to be closer to the observed prices than the other two models with these shorter maturity calls.

When considering the longer maturity call contracts from 2 months onward, the Black-Scholes implied options have an upward bias compared to the quoted prices, whilst the bias of the other two models is

not as large. The largest bias between the theoretical prices and observed prices is for at-the-money options.

Comparisons of put options show the theoretical option prices also biased downward from the observed option prices. The largest bias downwards is for the longer maturity out-of-the-money options. The two models with stochastic volatility do better with puts of longer maturity as they did with the call options. GARCH with systematic risk seems to do better than risk neutral GARCH alone for both calls and puts.

Satchell and Timmermann go on to explain why their results are somewhat different to Engle and Mustafa's:

Contrary to Engle and Mustafa this does not show, however, that the MSE of an option pricing model based on stochastic volatility produced a smaller MSE than the Black-Scholes option prices. This can be explained by two factors, First, this set of option contracts cover a wider range of maturities (1-12 months) than the data set analysed by Engle and Mustafa (up to four months). This may generate problems for the stochastic volatility model in attempting to choose values of the persistence parameters which give a good fit for both short and long maturities. The good fit for the contracts of longer maturity comes at the expense of the fit of the option contracts of shorter maturity. Secondly, since Engle and Mustafa's model with stochastic volatility was set in levels rather than log levels, their computations are not directly comparable to ours.

We do not calculate the MSE in our study here, but rather look at the traded option prices themselves to ascertain which model works better with each particular contract. This way, we can do away with aggregation problems associated with auto-quoting the Black-Scholes formula

The sensitivity of the systematic consumption risk model to the value of  $\mu$  is also an issue of the Satchell and Timmermann study. They find that by adjusting the risk premium from a low value ( $\mu = 11\%$ ) to a high value ( $\mu = 20\%$ ). A low risk premium implies a low correlation between consumption and stock prices. So with a low correlation the model incorporating systematic risk and stochastic volatility results in a lower overall bias than the other two models. In this manner, Satchell and Timmermann seem to deal with the mean-reversion process detailed in Amin and Ng. At values greater than 14% for  $\mu$ , the bias of the systematic consumption risk formula is greater than the other two models. Our study in this paper deals with the historical value of 14% for the return on the FTSE 100 index, which is the starting value of Satchell and Timmermanns. As we have put forth, we wish to test whether a higher value for the steady state volatility between two periods of differing steady state volatility produce a lower bias, and for which particular contracts this is the case.

In a more recent application of stochastic volatility in the pricing of options by Bollerslev (1996), he finds that:

The correct modelling of the long-run dependencies in the volatility process of the underlying asset may be as important as the choice of approximate option valuation method when pricing long maturity contracts.

Bollerslev provides a good account of EGARCH, IEGARCH and FIEGARCH modelling of long memory volatility on the S & P 500 index (as does Baillie (1998)), but assumes risk neutral valuation. Bollerslev accepts the notion of the absence of a closed form solution to the option pricing formula in the presence of time-varying volatility, but instead of using simulations, he uses Hull and Whites (1987) assertion that:

If the continuous-time volatility process is instantaneously uncorrelated with the aggregate consumption in the economy, the theoretical price of a call option is equal to the expected Black-Scholes price integrated over the average instantaneous variance during the life of the option.

The main point to Bollerslev's contention is that the longer the maturity of an option, the more susceptible is the price to the stochastic volatility.

It seems that the models he puts forth are the frontier in the analysis of stochastic volatility modelling.

It would be of interest to use these new models along with systematic consumption risk and to see whether the level of the risk premium affects the pricing errors between actual and derived option prices. This will be evident with further research.

By modelling the volatility of the underlying asset (The FTSE 100), as a GARCH process, we aim to test whether the prices of options reflect the increased persistence of volatility since the Asian Financial Crisis, under all three option pricing models described in this section. We stipulate that the only way that pricing errors converge to observed prices is if there is greater correlation between aggregate consumption in the economy and the return on the underlying asset. This correlation can only be incorporated if the Risk-Neutral assumption is relaxed.

### **III**

#### **Methodology.**

##### **Unit-Root Tests.**

The Stationarity of variables is one of the assumptions underlying Ordinary Least Squares (OLS) regression. This refers to the distributional moments of the time series being constant over time. (ie the mean and the variance). There is insurmountable evidence suggesting that both financial and economic data are non-stationary in their level form. In econometric terminology, this suggests the presence of a unit root.

A formal test for the presence of a unit root is the Augmented Dickey-Fuller (1979) procedure (ADF). If a series such as the Natural log of the FTSE 100 stock index has to be differenced in order to make the variables stationary then the series is said to be integrated of order 1 (I(1)). Natural logs of the series are taken to smooth out any outliers in the data. This will solve the problem of having large swings in the price level and therefore large percentage changes in the returns:

$$\log \frac{x_t}{x_{t-1}} = \log(x_t) - \log(x_{t-1})$$

The ADF procedure involves regressing the first difference of the series against a constant, the series lagged one period, the differenced series at  $n$  lags and a time trend:

$$\Delta y_t = \alpha_0 + \alpha_1 y_{t-1} + \sum_{i=1}^n \beta_i \Delta y_{t-i} + \gamma t + \varepsilon_t \quad (3.1)$$

where  $y_t = \log(x_t)$ .

Non-Stationarity is tested by the statistical significance of  $\alpha_1$  in equation (3.1). The following hypothesis is tested:

$H_0: \alpha_1 = 0$  (the null of a unit-root)

$H_1: \alpha_1 \neq 0$ ; (Alternative.)

The hypothesis that  $y$  is non-stationary is rejected if  $\alpha_1$  is significantly different from zero in equation (3.1). The null of a unit root is rejected if  $\alpha_1 \neq 0$ .

A decision must be made as to the lag length  $n$  and also whether a time trend is required. Generally when dealing with percentage changes a time trend is not required. These issues are dealt with in the results section.

The Phillips-Perron (1988) test can also be used to check for stationarity in the variables differenced form. In this alternative test for a unit-root, the hypothesis that  $\alpha_1 = 0$  is tested in the following equation:

$$\Delta y_t = \alpha_0 + \alpha_1 y_{t-1} + \mu + \varepsilon_t \tag{3.2}$$

The difference here being that the appropriate 'ag' length is set by the Newey-West (1987) procedure, where the t-statistic is corrected for the serial correlation in the error term via Newey-West. The transformation being:

$$(X'X)^{-1} \Omega (X'X)^{-1}.$$

The test statistic is the same as the ADF test statistic. The method of administration depends on the particular software package used. Both Eviews and Microfit were used in this paper (MicroFit results are shown in the appendix) and the results of the tests administered on the FTSE 100 data are discussed in the subsequent results section V.

Modelling the Variance.

Although modelling the mean may be important to ascertain certain empirical regularities in the data, we assume that a simple mean equation is enough to allow us to model the variance of the FTSE 100. In any case,



the impact of the return upon the price of an option is dealt with in other ways as described in the previous section. Furthermore, as put forward by Gannon (1996) and Nelson (1990) and later by McKenzie, Brooks (1999):

Results suggest that the order of the AR models has no real impact on the ARCH models estimated in continuous time.....McKenzie (1997) presents empirical findings for daily Australian bilateral exchange rate data which suggest that this same result may be found when considering data sampled in discrete time.

The discrepancies between continuous and discrete time models were alluded to in the previous section. Most economic models (including the one presented in this paper concerning intertemporal choice) are based upon discrete time processes, whereas financial type models (The Black-Scholes option pricing formula) are based on continuous time. Numerous authors have aimed to “bridge the gap to Continuous Time” (See Rossi (1996)), but many others, including Satchell and Timmermann (1995), put the issue to one side. The same is done in this paper although the problem is noted as a base for future research.

Our purpose is to only model the conditional variance as the variance of the underlying asset is a parameter, which does affect the price of an option written on this underlying asset. The mean equation can therefore simply be written as:

$$\Delta \log(x_t) = \beta_0 + \beta_1 \Delta \log(x_{t-1}) + \mu_t \quad (3.3)$$

where  $x_t$  is the index level of the FTSE 100 in this paper.

To test for ARCH Effects prior to the estimation of an equation for the conditional variance, a visual perusal of the change in the price level of the FTSE 100 sees evidence of the clustering of volatility. The usual approach is to look at the squared returns, although without looking at a graphical exposition of these squared returns, the change in the price itself in the time series is suffice to see these ARCH effects. Of particular importance is the period from the mid-1997 Asian financial crisis (see previous graph along with the graph of the log returns). This persistence of volatility is still present in mid 2000.

A more formal approach to testing for the clustering of volatility and therefore ARCH effects in the conditional variance of  $\mu_t$  in equation (3.3) is by hypothesis testing of the ARCH(q) specification for the conditional variance ( $h_t^2$ ) in the following equation:

$$h_t^2 = \rho_0 + \rho_1 u_{t-1}^2 + \rho_2 u_{t-1}^2 + \dots + \rho_q u_{t-q}^2. \quad (3.4)$$

Where:

$$H_0: \rho_1 = \rho_2 = \dots = \rho_q = 0 \quad \text{(The null of no ARCH effect.)}$$

Is tested against the alternative:

$$H_1: \rho_1 \neq 0, \rho_2 \neq 0, \rho_q \neq 0 \quad \text{(Alternative of an ARCH effect.)}$$

The test, commonly termed the Lagrange Multiplier (LM) test proposed by Engle (1982) involves running a regression of the squared OLS residuals from the mean equation (3.3) on the lagged squared residuals. The LM test will yield a statistic which should be above the 95 per cent critical value of  $\chi^2_1$  to reject the hypothesis that there are no ARCH effects in equation (3.3).

Once it is proven that ARCH effects are present, the specification of the conditional variance equation is carried out. We are interested in the variance of log returns in equation (3.3). The unconditional variance of  $u_t$  in equation (3.3) may well be constant ( $\sigma^2$ ) but the conditional variance  $h_t^2$ , may vary with time. The ARCH (1) model shows this process:

$$V(x_t | \Omega_{t-1}) = V(u_t | \Omega_{t-1}) = h_t^2 = \alpha_0 + \alpha_1 u_{t-1}^2$$

For this ARCH (1) model  $\sigma^2 = \alpha_0 / 1 - \alpha_1$  is the unconditional variance and is constant if  $\alpha_1 < 1$ .

The generalised autoregressive conditional heteroscedastic (GARCH(p,q)) model is used if the conditional variance takes the form of:

$$h_t^2 = \alpha_0 + \alpha_1 u_{t-1}^2 + \dots + \alpha_q u_{t-q}^2 + \varphi_1 h_{t-1}^2 + \dots + \varphi_p h_{t-p}^2 \quad (3.5)$$

The GARCH (1,1) model will equivalently be:

$$V(u_t / \Omega_{t-1}) = h_t^2 = \alpha_0 + \alpha_1 u_{t-1}^2 + \varphi_1 h_{t-1}^2 \quad (3.6)$$

The unconditional variance of  $u_t$  with a GARCH (1,1) specification will be given by  $\alpha_0 / 1 - \alpha_1 - \varphi_1$ .

For the unconditional variance to be positive (as one would expect),

The following restrictions are required:

$$1- \alpha_0 > 0$$

$$2- |\varphi_1| < 1$$

$$3-1-\alpha_1-\varphi_1 > 0$$

There are many different specifications of the GARCH (p,q) which can be estimated. In this paper we estimated upto GARCH(3,3) models for the two time periods of 1991-1997 and 1997-2000. Some of the models did not converge and thus their specification was excluded. It was important to note that when the GARCH(1,1) model was estimated for the entire period as a whole, there was no convergence suggesting that the shock due to the Asian Financial crisis has had much more permanent effects on the persistence of volatility than is evident previously in the 1990's.

Apart from specifications which do not converge, GARCH (p,q) models with insignificant t-statistics on the  $\alpha$  and  $\phi$  coefficients may be excluded along with negative parameters. The persistence in volatility will be shown by the sum of  $\alpha$  and  $\phi$ . This should be less than one, although it would seem very close to unity.

Satchell and Timmermann (1995) present the following GARCH (1,1) specification in their option pricing analysis:

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^q \alpha_i \sigma_{t-i}^2 \varepsilon_{t-i}^2 + \sum_{j=1}^p (\alpha_1 + \phi_j) \sigma_{t-j}^2 \quad (3.7)$$

On first glimpse it would seem that this model is a misprint , but when taken with respect to the option pricing formula they put forth, the specification merely represents the persistence of a shock in the form of the innovation  $\varepsilon_{t-1}^2$ . This innovation is random, they assume it is normally distributed. It should be noted that innovations evident in high frequency financial series data are more t-distributed than normally distributed (See Pagan (1996)). For simplicity we will also use normally distributed innovations in the process along the lines of Satchell and Timmermann. This aids our exposition but the innovations could be changed to reflect a t-distribution in future research. This would be the case if intra-day data were to be used.

Equation (2.24) of section II was used to extract a value of the preference parameter  $\kappa\sigma_c$  which is needed to ascertain the feedback of the rate of return parameter  $\mu$ , upon the option price. Monte Carlo simulations were used over the course of the daily returns of sample 1(1991-1997) and 2(1997-2000) to come up with the preference parameters for these periods. These preference parameters were then used in equation (2.25) and Monte Carlo simulations were used to derive the option price. The Specified GARCH process was used to show the persistence of the

previous periods innovation according to equation (3.7). We discuss Monte Carlo simulations next.

### The Monte Carlo Approach.

The assumption of risk neutrality where the equilibrium rate of return on every asset (inclusive of the underlying asset) is the risk-free rate was discussed in section II. The expected rate of return on an underlying asset can be shown to be:

$$E(S_T / S_t) = \exp(r[T-t]). \quad (3.8)$$

The return on the underlying will have a lognormal distribution with a mean of  $\exp r$ . Following Cox and Ross (1976), the generation of the distribution of stock prices in the period ahead is undertaken by the formation of random variables according to:

$$S_{t+1} = S_t \exp[r - \sigma^2/2 + \alpha x], \quad (3.9)$$

The random variable  $x$  is normally distributed with a zero mean and a unit variance. Satchell and Timmermann (1995) also generate normally distributed random variables in simulations. As was explained, the evidence of t-distributed returns in high frequency data is apparent in the

literature, see Pagan (1996). As explained, there is an avenue for further research in the generation of t-distributed random variables.

The Monte Carlo approach to the generation of random variables in scenario analysis dates back to the Manhattan Experiments and relevant applications in the physical and biological sciences.. Boyle (1977) discusses the application of Monte Carlo simulation to option pricing. This approach is used when a closed form solution such as the arbitrage solution in the Black-Scholes formula, is not available, as discussed in section II.

Although our explanation of the Monte Carlo approach is as a definite integral here, the application is relevant in a discrete time framework as well. Let  $g(y)$  be an arbitrary function and  $f(y)$  a probability function so that the integral of it is equal to one. An estimate of  $g$  below:

$$\int_A g(y)f(y)dy = g. \quad (3.10)$$

Is obtained from  $n$  sample values of  $y_i$ . The values of  $y_i$  are taken from the probability function defined in  $f(y)$ . The estimate of  $g$  will be the average over the generated samples so that:

$$G = 1/n \sum_{i=1}^n g(y_i) \quad (3.11)$$



The standard deviation of the estimate is given in the usual fashion as:

$$s^2 = 1/(n-1) \sum_{i=1}^n (g(y_i) - G)^2 \quad (3.12)$$

As Boyle contends, with a very large number of simulations, the distribution will mimic that of a normal distribution. The standard deviation of the estimated  $g$  will be equal to  $s/n$ . The confidence intervals can therefore be reduced by increasing the amount of simulations generated. By increasing the number of simulations by 100, we decrease the standard deviation ten-fold.

With increasing computer power, one would expect that increasing the amount of simulations would be an easily achieved task. A more efficient approach would be to decrease the confidence limits by attacking the standard deviation of the estimate itself ( $s$ ). This is the purpose of approaches such as Hammersley and Handscomb (1964) and particularly the Control Variate method to which we now turn our attention.

The basic idea underlying this method is to replace the problem under consideration by a similar problem which has an analytical solution. The solution of the simpler problem is used to increase the accuracy of the solution to the more complex problem.

Boyle, (1977).

If we can analytically evaluate an integral such as:

$$\int g(y)h(y)dy = G^I \quad (3.13)$$

where  $h$  (the control variate) is now the probability function whereas  $f$  in equation 1 before, could not be evaluated analytically, we can evaluate the following integral by “crude Monte Carlo methods”:

$$G = G^I + \int g(y)[f(y)-h(y)]dy \quad (3.14)$$

The reduction in the variance of  $g$  as compared to the variance in  $G$  is the efficiency measure of the control variate. This in turn will depend upon how closely  $h$  models the behaviour of  $f$ , which is most of the time inversely related to the ease evaluation. There is therefore a tradeoff.

A more intuitive approach to Monte Carlo simulations in discrete time models is provided in Ravindran (1996). The basic idea is to simulate the underlying asset path starting at your point of reference (time 0) and ending at the contract expiry date. At the terminal date (expiry date) the option will have a particular pay-off. The payoff will only be non-negative if the underlying asset price is greater than the exercise price. This simulation is carried out several thousand times (most applications use 10,000 simulations). The average value of this pay-off is then brought to the present by the continuous rate of return to give the present value of

the option. This will then be the premium paid for the option. Of importance is the type of innovation assumed in the simulation. We use normally distributed innovations here in assuming that returns on the FTSE 100 are normally distributed, but as we have mentioned a few times prior, high frequency returns seem to be more t-distributed. This process can be carried out in software packages such as RATS, although we (along with Ravindran) use MicroSoft Excel.

Satchell and Timmerman (1995) use the simulated value of the Black-Scholes option price as the control variate to improve the precision of their Monte Carlo simulations.

$$O^{BS} = X_0 \exp(-r_f t) \times E[\text{Max}(\exp(t(r_f - \sigma^2/2) + \sigma \sum_{j=1}^t \varepsilon_j) - K/X_0, 0)] \quad (3.15)$$

$$O = X_0 \exp(-r_f t - (\sigma_c^2 \kappa^2 t)/2) \times E[\exp(-\kappa \sigma_c \sum_{j=1}^t \varepsilon_j) \times \text{Max}(\exp(t\mu + \sum_{j=1}^t \sigma_j \varepsilon_j) - K/X_0, 0)] \quad (3.16)$$

Calculating the control variate option price estimation involves the following formulation in the same manner as equation (3.14) of Boyles exposition in continuous time:

$$O^{SCG} = O - q(O^{BS} - O_{actual}^{BS}) \quad (3.17)$$

The  $q$  in equation (3.17) refers to  $\text{Cov}(O, O^{BS})/\text{Var}(O^{BS})$ . Equation (3.16) is simply our theoretical option values under systematic consumption risk using GARCH and equation (3.15) are the simulated Black and Scholes option prices. Notice the value of zero for  $\kappa$  and the constant variance  $\sigma$ .  $O_{actual}^{BS}$  are the theoretical values from the Black-Scholes formula.

The options prices for the three option pricing models described in section II are presented in the next section.

The option prices are calculated for two separate time periods. The second time period (starting from the 7<sup>th</sup> January 1997) is relevant as it displays an increase in the persistence of volatility. We are interested in which model of theoretical option prices most closely replicates the quoted option prices on the FTSE 100 Stock Index. In this manner, we can directly test Satchell and Timmermans (1995) exertion that the reason their option prices under systematic consumption risk and GARCH were so biased, in comparison to the other two models, is because of the low value of the steady state variance:

Once the steady state variance of the models with stochastic volatility is close to the constant variance of

the Black-Scholes model, the errors of the three models converge.

Satchell and Timmerman(1995)

## **IV**

### **Data.**

This paper estimates two separate GARCH equations for the conditional variance over two sample periods in determining theoretical option prices and comparing these with actual prices for traded options on the underlying FTSE 100 stock index. The first sample period was from the 1<sup>st</sup> of July 1991 to the 7<sup>th</sup> of January 1997 (1485 observations) and the second was from the 8<sup>th</sup> January 1997 to the 17<sup>th</sup> March 2000 (865 observations). Daily stock index data on the FTSE 100 was taken from Datastream International. We don't believe that the difference in the amount of observations between the two sample periods will have any bearing on the results, in any case it was the latest data we could retrieve in capturing the structural change of the Financial Crisis during 1997.

LIFFE DATA Int. provided option prices on the FTSE 100 for the 7th of January 1997 and equivalently for the 17th March 2000. The daily price of each contract is the average price of the particular traded option on the market that day. Our analysis differs to Satchell and Timmermann (1995) in that we are only interested in traded options rather than the entire spectrum of contracts on offer. As Satchell and Timmermann put it:

Although the majority of the contracts thus had an open interest, one has to interpret the results with caution because of the dangers associated with mis-pricing of quoted but non-traded options. This is particularly important in the present case since some of the price quotes were based on an 'auto-quote' procedure whereby option contracts that were not actually traded on a given day had a price quoted by plugging an estimate of the market volatility into the Black-Scholes formula. This procedure is likely to bias the comparisons of various option pricing formula's towards the Black-Scholes.

Our aim is to compare the pricing biases in the theoretical option prices as compared to actually traded options in the two separate sample periods. By looking at only traded options, we believe this bias towards the Black-Scholes may be overcome somewhat. In any regard, the biases that Satchell and Timmermann note, place the blame upon the bias toward Black-Scholes.

The price Level of the FTSE 100 Stock index on the 7<sup>th</sup> January 1997 was 4078.8, and on the 17<sup>th</sup> March 2000, it was 6557.9. The options traded are European Style options, which are only exercisable on the expiry date. The expiry date for FTSE 100 stock options is the last trading day, which is the third Friday of the expiry month. In the event that the last trading day not being a trading day then the previous business day will be the last trading day. The unit of trading is valued at 10 pounds per index point. So for example, the value of the stock index on the 7<sup>th</sup> January 1997 is

40,788.00 pounds. The minimum tick movement is 0.5 equivalent to 5 pounds. Additional exercise prices are introduced on the business day after the underlying index level has exceeded the second highest, or fallen below the second lowest, available exercise price.

The exchange delivery settlement price (EDSP) is based on the average level of the FTSE 100 Index between 10.10 and 10.30 on the last trading day.

Of interest is the fact that the 17<sup>th</sup> January 2000 seems to be the last trading day the contract expiring in March. There is therefore a heavy turnover in the volume of trades. Options may be settled on this day and rolled over to subsequent expiry months. This is usually the case with hedgers, especially fund managers.

On total there were 17 call and 13 put options traded on the 7<sup>th</sup> January 1997. Whilst there were 40 call and 32 put options traded on the 17<sup>th</sup> March 2000. The extra amount of trades in the latter sample period seems due to the particular trading day as explained above. Needless to say, this should still not affect our analysis in comparing pricing biases between the two sample periods although not having longer maturity dates for the first sample period will prove troublesome for any comparisons made.



The risk free rate on the 7<sup>th</sup> January 1997 was 6% and on the 17<sup>th</sup> March 2000 it was 6.18%. The historical expected return on the FTSE 100 was taken from Satchell and Timmermann as being 14% pa. The constant variance for the Black and Scholes calculations is discussed in the next section as well as the initial variance for the persistent volatility models.

With the above hurdles in mind, the data collected here is used to compare the accuracy of our model with Systematic Consumption Risk using GARCH, Engle and Mustafa's (1992) model under risk neutrality using GARCH and the celebrated Black and Scholes (1973) option pricing formula with constant variance.

## V

### Results.

#### Unit Root Tests.

In testing the null hypothesis of a unit root, the Augmented Dickey Fuller (ADF) tests outlined in section III failed to reject the Null of a unit root in the level form of the FTSE 100 stock index with a linear trend. When the log of the price level was taken and the variables were in differenced form, the Null of a unit root is rejected and the variables are stationary in their differenced log form. (ie. Log of the returns).

The Phillips Perron test for stationarity in differenced form also rejects the null of a unit root in log differenced form.

**Table 5.1:**

#### **Level Form ADF Tests**

<i>DF</i>	-2.9235
<i>ADF(1)</i>	-3.1383
<i>ADF(2)</i>	-2.9593
<i>ADF(3)</i>	-2.7825
<i>ADF(4)</i>	-2.7577
<i>ADF(5)</i>	-2.7170

**95% Critical value for the Augmented Dickey-Fuller statistic = -3.4143**

**Table 5.2:**

**Log Difference Form ADF Tests**

<i>DF</i>	-44.7003
<i>ADF(1)</i>	-34.7401
<i>ADF(2)</i>	-29.5321
<i>ADF(3)</i>	-25.2374
<i>ADF(4)</i>	-22.6455
<i>ADF(5)</i>	-21.8740

95% Critical value for the Augmented Dickey-Fuller statistic = - 2.8633

**Table 5.3:**

**Phillips Perron Tests For Unit Root.**

<i>T-Ratio</i>	<i>[Prob]</i>
3.8756	[.000]

**Testing for ARCH effects**

Even without looking at the squared returns, the clustering of volatility is evident in our graph of the change in price level of the FTSE 100 stock index (See depictions in introduction). This is substantiated by the graph of the log returns. The persistence of volatility is sharpest after the Asian Financial crisis in 1997. This is why we wish to compare options prices prior 1997, and during and after 1997, with the latest data on option prices for traded options on the FTSE 100.

As well as a visual inspection, the formal Lagrangian Multiplier test for the presence of ARCH effects is used. Lags from 1 period to 12 periods were taken for the entire sample period. The LM test results provide conclusive evidence of the presence of ARCH effects in the entire sample.

Table 5.4:

LM Statistic	
<i>CHSQ(1)</i> =	46.6498[.000]
<i>CHSQ(2)</i> =	98.5602[.000]
<i>CHSQ(3)</i> =	146.0265[.000]
<i>CHSQ(4)</i> =	159.4462[.000]
<i>CHSQ(5)</i> =	174.6721[.000]
<i>CHSQ(6)</i> =	189.4350[.000]
<i>CHSQ(7)</i> =	205.9993[.000]
<i>CHSQ(8)</i> =	212.4321[.000]
<i>CHSQ(9)</i> =	212.8332[.000]
<i>CHSQ(10)</i> =	235.0355[.000]
<i>CHSQ(11)</i> =	250.8043[.000]
<i>CHSQ(12)</i> =	269.3286[.000]

**Univariate GARCH Modelling**

We present GARCH specifications for both sample periods right up to a GARCH (3,3) specification. Some models did not converge in MicroFit. Reasons for this were presented as restrictions on the coefficients in section III. It was interesting to note that there was no convergence when a GARCH (1,1) model was fitted for the entire sample period. We believe

the reason for this to be the fact that the shock to the system of the financial crisis seems to have had permanent effects. (an IGARCH process). This has lead to the contravening of the restrictions mentioned and thus no convergence.

**Table 5.5:**

<b>Period 1 (1<sup>st</sup> July 1991 - 7<sup>th</sup> January 1997)</b>	
<i>Persistence (<math>\alpha + \beta</math>)</i>	
<i>GARCH (1,1)</i>	$0.059655 + 0.90471 = 0.964365$
<i>GARCH (3,2)</i>	$0.078224 + 0.89802 = 0.976244$
<i>GARCH (3,3)</i>	$0.059435 + 0.93087 = 0.990305$

**Table 5.6:**

<b>Period 2 (8<sup>th</sup> January 1997 - 17<sup>th</sup> March 2000)</b>	
<i>Persistence (<math>\alpha + \beta</math>)</i>	
<i>GARCH (1,1)</i>	$0.052604 + 0.92488 = 0.977484$
<i>GARCH (2,1)</i>	$0.071812 + 0.87152 = 0.943332$
<i>GARCH (1,2)</i>	$0.058209 + 0.91504 = 0.973249$
<i>GARCH (1,3)</i>	$0.057917 + 0.91336 = 0.971277$
<i>GARCH (2,2)</i>	$0.079886 + 0.87314 = 0.953026$
<i>GARCH (2,3)</i>	$0.080874 + 0.85242 = 0.933294$
<i>GARCH (3,3)</i>	$0.098293 + 0.81593 = 0.914823$
<i>GARCH (3,2)</i>	$0.10212 + 0.79738 = 0.899500$

GARCH specifications were also estimated for t-distributed errors, and are presented in the appendix. As our simulations involved the generation of normally distributed errors in the mean equation we only need the

GARCH models with normally distributed errors. As we mentioned, the simulation' with t-distributed errors is left for future research.

According to McKenzie and Brooks (1999):

Unfortunately, neither economic nor econometric theory provides much in the way of guidance for the selection of an optimal model from among those fitted. Apart from the excluded models which did not converge. The common approach is to also exclude coefficients with insignificant t-ratio's. Other than this, the choice of the model is left at the researchers discretion. The disadvantage of using MicroFit as the software of choice in our analysis here is that it is pretty much limited to the (1,1) model. For this purpose we are led to accept the GARCH (1,1) As the specification of choice. Compared to All of the previous studies, our values for the persistence parameters are by far the highest. Satchell and Timmermanns results showed a persistence of volatility from one period to the next of only 0.92 whilst the results of Duan showed modest persistence of only 0.72. Engle and Mustafa's persistence was also in the high 0.85 bracket. This is clear evidence that returns today are indeed much more volatile. The longer lagged results are open to interpretation. Maybe because we have a smaller number of observations in the second period, the persistence is greater. This is yet more reason for further research.

The most evident observation in the results of the study so far is that the volatility of the returns does indeed show an integrated GARCH process. So yet another avenue for future research in options pricing is open by modeling the volatility process as IGARCH and FIGARCH processes and

even further on as FIEGARCH etc following Bollerslev (1996) and Baillie (1998). For such advanced models, the use of software such as GAUSS and MATLAB is preferable.

### Option Prices.

An example of our Monte Approach is given below for the Black-Scholes case of constant variance. The constant standard deviation (needed for this calculation) of the returns for the period 4<sup>th</sup> July 1991 to 6<sup>th</sup> January 1997, was calculated at 0.007464. While the value for the second sample period 8<sup>th</sup> January 1997 to 17<sup>th</sup> March 2000 was 0.01157.

7th January Call Option January 1997 Exercise 3975				
Time	FTSE 100	Random Generation	Return = $(1+r)+(st\ dev * Random)$	3975 FTSE 100
0			4078.8	4078.8
1	4078.8	-0.126751729	0.999293925	4075.920062
2	4075.9201	0.721609013	1.003312097	4089.419905
3	4089.4199	-1.089881607	1.002850006	4101.074778
4	4101.0748	-0.604259185	1.006199297	4126.498558
5	4126.4986	1.931612132	1.001081011	4130.959347
6	4130.9593	-0.125210136	1.002172615	4139.934333
7	4139.9343	-1.266002982	1.005599585	4163.116247
Risk Free Rate      Standard Deviation				
			0.00024	0.007464
Value at t = 7			Value at t = 0	
188.1162471			187.8005149	

This represents a simulation of a January-1997 call option on the FTSE 100 with an exercise price of 3975. The value of the option on the 7<sup>th</sup> of January 1997 was observed to be 117 whilst our simulated value here was 187.8 (bottom right). This simulation was run 10,000 times to achieve an

average result of 157.5 which is much higher than what is observed in the market. The same approach is followed to find the GARCH option prices under risk neutrality and then the GARCH with systematic consumption risk prices.

For the GARCH<sup>11</sup> approach without systematic consumption risk, an algorithm which translates the GARCH implied persistence of volatility from one period to the next was designed in EXCEL and the same Monte Carlo simulations were run as according to the Black-Scholes approach just outlined.

The GARCH approach with systematic risk was somewhat more difficult. The estimated parameters for the value of  $\kappa\sigma_c$  were 0.032 for the period from 4<sup>th</sup> July 1991 to the 7<sup>th</sup> January 1997, and 0.052 for the period from the 8<sup>th</sup> January 1997 to the 17<sup>th</sup> March 2000. These results should be viewed with caution as our values for the remaining parameters are taken from the previous literature and may not be the wisest choice. But clearly there seems to be a higher correlation between consumption and mean returns. Coupled with a higher value for the steady state volatility for the second sample period.  $(0.1172/1-0.977) = 5.0956$ , Compared to  $(0.007464/1-0.9643)= 0.209$ . According to Satchell and Timmermann, the pricing errors should converge.



Considering the call options first. On the 7<sup>th</sup> of January 1997, it is evident that the three options pricing models overestimate the observed prices that are in the money. The Black Scholes has the highest pricing errors followed by the GARCH model under risk neutrality and then the GARCH model with systematic consumption risk. In the money options, refer to the exercise price being lower than the spot price in the case of calls, and vice-versa in the case of puts. As the exercise price moves out of the money, the options prices from our three models fall faster than the observed prices so that generally the option pricing models underestimate option prices which are far out-of-the-money. Of note is the fact the Black-Scholes model slightly overestimates at the money options but does a good job for far out-of-the-money options.

When the term to maturity increases as is evident from the different expiry months, our three option pricing models diverge for in-the-money options. The GARCH model with systematic consumption risk is still the least biased in regard to pricing errors for these in-the-money options and consistently underestimates at-the-money and in-the-money options with increasing terms to maturity.

By looking at the call option prices observed and calculated on the 17<sup>th</sup> March when the steady state volatility has increased substantially, we can compare the bias of each pricing model and test the assertion that the prices of all three models converge. There is slight convergence apparent with increasing maturity although the December Call options cannot justify this on closer inspection.

Of particular note is that the GARCH model with systematic consumption risk consistently underestimates option prices which are in-the-money, with increasing maturity. The Black-Scholes model once again overestimates option prices by far more than the other two models for options which are in-the-money and slightly underestimates out-of-the-money call options.

The results of the contracts for put options on the 7<sup>th</sup> January 1997 are mixed. For January expiry, both our models overestimate the option price, and the Black-Scholes underestimates the option price. Surprisingly, this is reversed in the put option contracts for February expiry. It seems in this case that there may be other relevant information acting upon the option market in this case. Regardless, our results for put options here are severely hampered because of the lack of longer maturity options data on this date and this should be taken into regard when

making comparisons between the two time periods. Our results are not consistent in that having the Black-Scholes model value options lower than the GARCH models in January puts, and then vice versa for the February puts is hard to explain. All three models show conclusive evidence of convergence in the second sample period where the steady state volatility is vastly increased.

Moving to the Put options on the 17<sup>th</sup> March 2000. The Black Scholes model clearly under estimates option prices for in-the-money options. This is even more apparent with increasing maturity. Both GARCH models under risk neutral valuation and systematic consumption risk overestimate out-of-the-money put options. GARCH and consumption risk does a particularly bad job with increasing maturity. Black and Scholes is very precise with out-of-the-money puts.

From a technical point of view, there seems to be room for an intertemporal pricing model in the options valuation tool-kit. The GARCH model with systematic consumption risk outperforms both the other two models for in-the-money call options of increasing maturity although there is a downward shift when the steady state volatility increases as in the case of in-the-money call options in our second sample period. This leads the consumption risk model to underestimate in the

money options when there is a higher correlation between consumption and returns. Secondly, and in agreement with Satchell and Timmerman, the Black-Scholes model does a particularly good job of pricing put options and options that are just out-of and out-of the money.

Coming up with an intuitive explanation for the regularity just identified can come from many angles. A higher correlation between consumption and returns on the index, means that the drift parameter  $\mu$  has a greater feedback upon the option price. So higher returns could possibly dampen the increase in the steady state apparent in the second sample period thus producing a lower value for the options priced under systematic consumption risk and GARCH.

The Black-Scholes model , and to a certain extent the GARCH model under risk neutralization still seems to over-value long-term maturity calls and slightly undervalue puts. When writing contracts, holders of the underlying security, say pension funds who would hold stocks in their portfolio, would be attaining a premium upon what the market would value these instruments. The performance of these portfolios would be vastly improved.

If the persistence is an integrated process, in that shocks last for longer periods of time, the GARCH models' prices will converge with those of the Black and Scholes. Our measures of persistence in both periods differs ever so slightly so we would not expect a huge convergence nor do we see one.

Our results, apart from confirming some already noted regularities about option pricing, prove that the convergence of the three option pricing models with observed option prices, when there is increasing steady state volatility is a slow and unreliable process. A higher level of volatility is subdued by a higher correlation between consumption and mean returns, so that our GARCH model with systematic consumption risk underestimates call options and slightly overestimates put options which are in-the-money. The effects on out-of-the-money options are found to be negligible.

We must also note the limitations of our study with respect to the synchronisation of the option prices with that of the underlying FTSE 100. We believe that because of the large number of observations of the FTSE 100, the use of the average option price on the day will have a minimal pricing bias effect. In any regard the difference between the lowest and highest option prices on both days did not seem to large so we

have continued in the same way that Satchell and Timmerman (1995) have proceeded. But this can also open an avenue for further research in investigating how the prices are biased if the lowest and highest prices on the day are compared to the average price on the traded option that day.

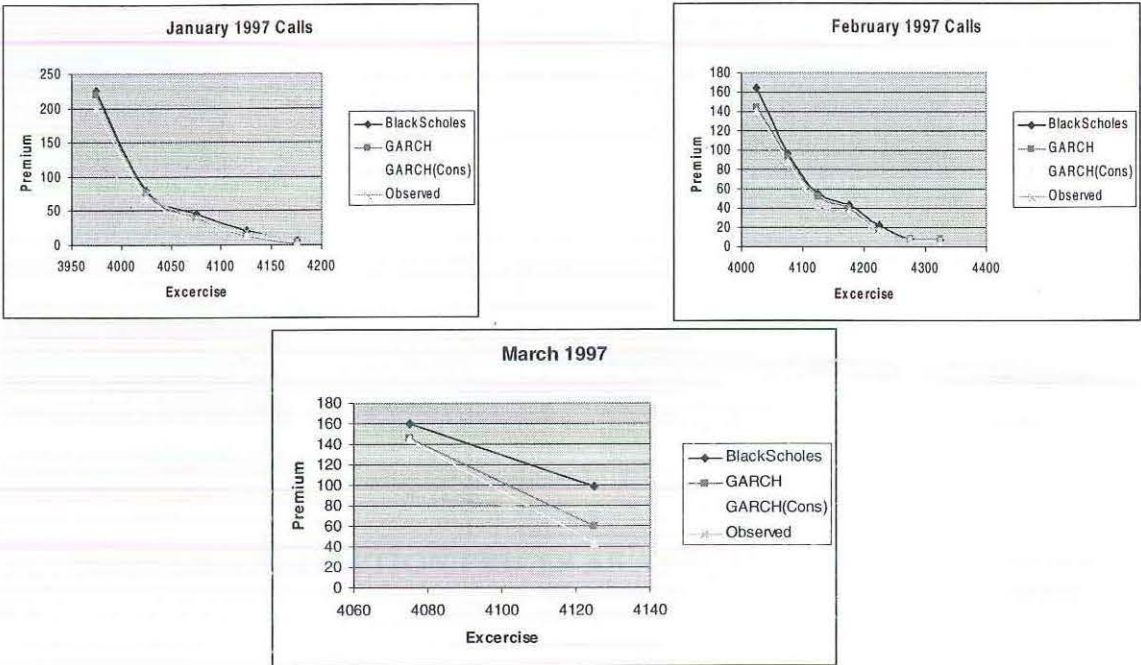


Figure 5.1: CALL OPTIONS AS AT 17<sup>TH</sup> MARCH 1997.

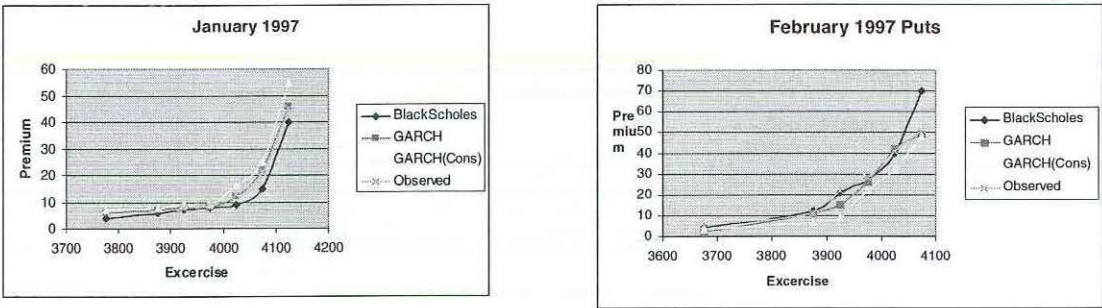


Figure 5.2: PUT OPTIONS AS AT 17<sup>TH</sup> MARCH 1997

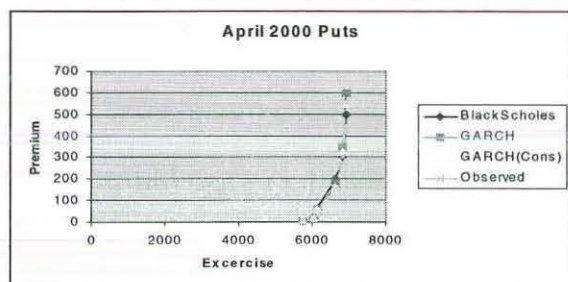
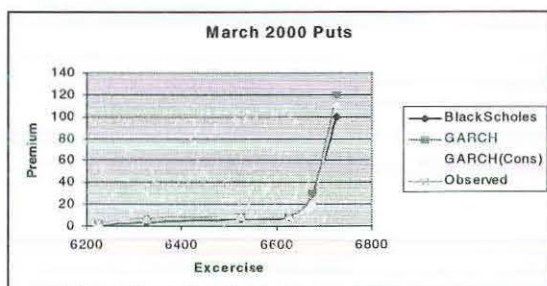
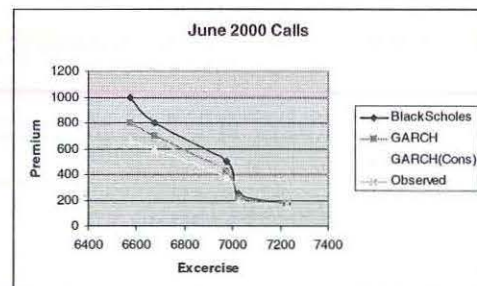
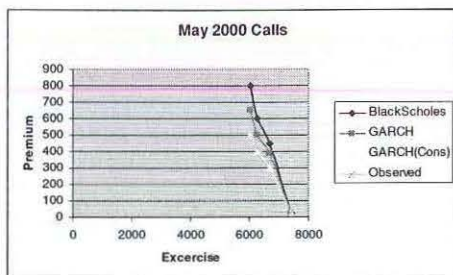
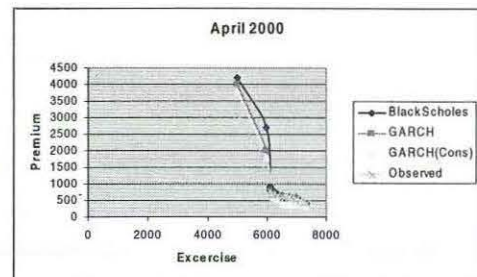
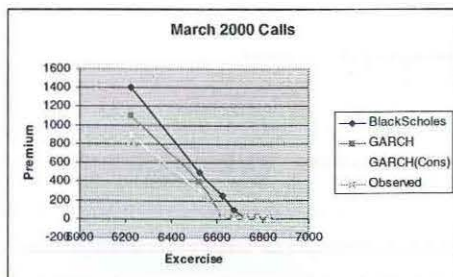


Figure 5.3: CALL OPTION PRICES AS AT 7 JANUARY 2000

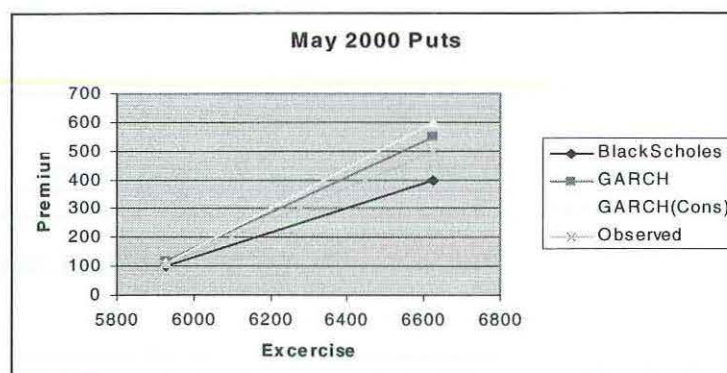


Figure 5.4: PUT OPTIONS AS AT 7 JANUARY 2000.

## VI

### Summary and Conclusions.

The complexity and monotony of option pricing when no closed form solution exists in a discrete time framework, is probably the main reason for the Black-Scholes popularity. Modelling investor behaviour is simple if everyone in the market is using the same formula and this may be why option values which are close to or out-of-the money closely resemble the Black-Scholes implied option prices.

This paper describes three different conceptual option pricing models, and looks at their performance with respect to different attributes on the underlying asset the option is written on. If the steady state volatility of the underlying asset increases then the implied option prices from the Black Scholes model, the Risk Neutral model with GARCH, and the Consumption risk model with GARCH should theoretically converge to the observed option price if holders of the option take the persistence of the underlying assets volatility into account. Our results show that this convergence process is ever so slight and the effect of an increase in the persistence of volatility has little if any effect upon the pricing biases apparent in the models.



Another finding is that if the correlation between consumption and the mean return on the underlying increases, there is a downward shift in call option prices implied by the GARCH model with systematic consumption risk. Although the effect on the put options is questionable. This means that the correlation between consumption and returns is an important factor in the pricing of models. The particular value for this parameter should be the focus of future research.

The variance of the underlying governs the chances that the contract will expire in the money. The larger the variance, the greater this probability and therefore the more valuable the option. The contradiction lies in the fact that our models continually overprice call options that are far in-the-money.

Our aim was to find which pricing model produced the closest option prices to the observed option prices and to see whether the introduction of systematic consumption risk improves the precision of an option pricing model. GARCH model with systematic consumption risk does a better job than the other two models with options that are far in-the-money and of longer maturity. The Black-Scholes is preferred at-the-money and out-of-the-money. This tells us that holders of options undervalue the expectation that their options will finish in-the-money when the options

are already trading in-the-money. They believe that the variance from the previous period will continue to the next period and this may increase the chances that the option may fall out-of-the-money at expiry. This may be the case if the persistence is an integrated process and this should be an impetus for further research using the new IGARCH and FIGARCH models put forward in the recent volatility literature.

## **References.**

- Amin. K, Ng. V, "Option Valuation with Systematic Stochastic Volatility", *Journal of Finance*, N 3, July 1993, 881-910.
- Baillie. T, "Modelling long memory volatility in asset returns", *Derivatives Use, Trading and Regulation*, V 4, No 3, 1998, 233-243.
- Baillie. T, "Long memory processes and fractional differencing", *Journal of Econometrics*, V 73, 1996, 5-59.
- Baillie. T, Bollerslev. T, Mikkelsen. H, "Fractionally integrated Generalized Autoregressive Conditional Heteroskedasticity", *Journal of Econometrics*, V 74, 1996, 3-30.
- Black. F, Scholes. M, "The pricing of options and corporate liabilities", *Journal of Political Economy*, V 81, 1973, 637-659.
- Bollerslev. T, "Generalized autoregressive conditional heteroskedasticity", *Journal of Econometrics*, V 31, 1987, 307-327.
- Bollerslev. T, "Modelling and pricing long memory in stock market volatility", *Journal of Econometrics*, V 73, 1996, 151-184.
- Boyle. P, "Options: A Monte Carlo Approach", *Journal of Financial Economics*, V 4, 1977, 323-338.
- Brennan. M, "The Pricing of Contingent Claims in Discrete Time Models", *Journal of Finance*, V 1, March 1979, 53-71.
- Cox. J, Ross. S, "The valuation of options for alternative stochastic processes", *Journal Financial Economics*, V 3, 1976, 145-166.
- Duan. J, "The GARCH Option pricing model", Mimeo, Faculty of Management, McGill University, 1993.
- Engle. R, "Autoregressive conditional heteroskedasticity with estimates of the variances of U.K. inflation, *Econometrica*, V 50, 1982, 987-1008.
- Engle. R, Mustafa. C, "Implied ARCH models from Option prices", *Journal of Econometrics*, V 52, 1992, 289-311.

Hammersley.J, Handscomb. D, "Monte Carlo methods", 1964,(Methuen, London).

Hull. J, White. A, "The pricing of options on assets with stochastic volatilities", *Journal of Finance*, V 42, 1987, 281-300.

Ingersoll. J, "A theoretical and Empirical Investigation of the Dual Purpose Funds, An application of Contingent Claims Analysis", *Journal of Financial Economics*. V 3, 1976, 83-123.

McKenzie. M, Brooks. R, "Research Design issues in Time-Series Modelling of Financial Market Volatility", V 2, 1999, McGraw-Hill.

Merton. R, "Theory of rational options pricing", *Bell Journal of Economics and Management Science*, V 4, 1973, 141-183.

Merton. R, "Option Pricing when underlying Stock Returns are Discontinuous", *Journal of Financial Economics*, V 3, 1976, 125-144.

Nelson. D, in "Modelling Stock market Volatility", 1996, Academic Press.

Newey. W, West. K, "A simple positive semi-definite heteroskedasticity and autocorrelation consistent covariance matrix", *Econometrica*, V 55, 1987, 703-708.

Pagan. A, "The econometrics of financial markets", *Journal of Empirical Finance*, V 3, 1996, 15-102.

Ravindran. K, "Monte Carlo simulations: Part 1", *Derivatives Use, Trading and Regulation*, V 2, N 1, 1996, 104-109.

Ross. C, "Black-Scholes and Beyond", 1993, McGraw-Hill.

Rubenstein. M, "The valuation of uncertain income streams and the pricing of Options", *Bell Journal of Economics and Management Science*, V 7, 1976, 407-425.

Sarkar. S, "Applications of option theory in managerial decisions", *Derivatives Use, Trading and Regulation*, V 2, N 1, 1996.

Satchell. S, Timmermann. A, "Option pricing with GARCH and systematic consumption risk: Part One", *Derivatives Use, Trading and Regulation*, V 1, No 3, 1995. 279-287.

Satchell. S, Timmermann. A, "Option pricing with GARCH and systematic consumption risk: Part Two", *Derivatives Use, Trading and Regulation*, V 1, No 4, 1995, 353-367

Smith. C, "Option Pricing: A review", *Journal of Financial Economics*, V 3, 1976, 3-51.

Smith. C, " Alternative Methods for Raising Capital: Rights Versus Underwritten Offerings", *Journal of Financial Economics*, V 5, 1977, 273-307.

Wiggins. J, "Option values under stochastic volatility: Theory and empirical estimates", *Journal of Financial Economics*, V 19, 1987, 351-372.